The Block Point Process Model for Continuous-Time Event-Based Dynamic Networks

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Statistical Inference for Network Models
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Continuous-Time Event-Based Dynamic Networks

- Relational event data with **fine-grained** timestamps
  - Facebook wall posts (Viswanath et al., 2009)

- Represent events as triplets \((i, j, t)\)

- Goal: build statistical model for these relations over time

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<th>Timestamp</th>
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Models for Static Networks

- If we discard timestamps, events become edges \((i, j)\) in a static network.
- Represent network by \(N \times N\) adjacency matrix \(A\).

**Stochastic block model (SBM):**

- Latent classes:
  - 4
  - 35
  - 97

- Edge probabilities between classes:
  - 0.1 0.9 0.1
  - 0.1 0.1 0.9
  - 0.9 0.1 0.1
Models for Discrete-Time Dynamic Networks

- If we aggregate events over time windows, we get a discrete-time snapshot-based network representation.

- Discrete-time SBMs (Yang et al., 2011; Xu and Hero, 2014; Xu, 2015; Matias and Miele, 2016)

- Trade-offs in choosing snapshot length:
  - Too long: loses temporal resolution
  - Too short: increases number of snapshots and causes model to forget too quickly due to short-term memory
The Block Point Process Model (BPPM)

• Our approach: Model event triplets \((i, j, t)\) directly using SBM-like generative structure
  – Divide nodes into \(K\) classes forming \(p = K^2\) blocks (assuming directed events)
  – Generate times of events in each block using a point process model
  – Randomly associate event with a pair of nodes \((i, j)\) in the block (thinning)
  – We use an exponential Hawkes process model in practice
Our Contributions

• We prove that static networks resulting from the BPPM follow an SBM as $N \to \infty$
  – We provide an upper bound on the deviation from independence for finite $N$

• We develop a principled inference procedure for the BPPM using local search initialized by spectral clustering
  – Scales to 5,000+ nodes and 100,000+ events

• We demonstrate that the BPPM is superior to discrete-time network models regardless of snapshot length
Comparison with Discrete-Time SBM

- Prediction task: Attempt to predict time to next event (Facebook wall post) in each block
  - 3,586 nodes and 137,170 events in data set

![Graph showing MSE vs. Length of time snapshot for Discrete-time SBM and Block Hawkes model]
Relationship to SBM

- Identical distribution of adjacency matrix entries within block satisfied by BPPM generative procedure
- But independence of entries is not satisfied!
  - Denote deviation from independence by
    \[
    \delta_0 = \Pr(a_{ij} = 0 | a_{i'j'} = 0) - \Pr(a_{ij} = 0) \\
    \delta_1 = \Pr(a_{ij} = 0 | a_{i'j'} = 1) - \Pr(a_{ij} = 0)
    \]

Theorem (Asymptotic Independence Theorem). Consider an adjacency matrix \(A\) constructed from the BPPM over some time interval \([t_1, t_2]\). Then, for any two entries \(a_{ij}\) and \(a_{i'j'}\) both in block \(b\), the deviation from independence given by \(\delta_0, \delta_1\) defined in (1) is bounded in the following manner:

\[
|\delta_0|, |\delta_1| \leq \min \{1, \mu_b/n_b \}
\]

where \(\mu_b\) denotes the expected number of events in block \(b\) in \([t_1, t_2]\), and \(n_b\) denotes the size of block \(b\). In the limit as the block size \(n_b \to \infty\), \(\delta_0, \delta_1 \to 0\) provided \(\mu_b\) is fixed or growing at a slower rate than \(n_b\). Thus \(a_{ij}\) and \(a_{i'j'}\) are asymptotically independent in the block size \(n_b\).